(7 pages) **Reg. No. :** .....

Code No. : 30936 E **Sub. Code : FCMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2024

First Semester

Mathematics — Core

ALGEBRA AND TRIGONOMETRY

(For those who joined in July 2024 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. _____ is a factor of $x^5+4x^4+3x^3+3x^2+4x+1=0$.

(a) $x-1$ (b) $x+1$

(c) $x-\frac{1}{2}$ (d) $x+\frac{1}{2}$

2. A reciprocal equation $a_0x^n+a_1x^{n-1}+....+a_n=0$ is said to be of first type if _____.

(a) $a_{n-r}=a_r$ (b) $a_{n-r}=-a_r$

(c) $a_{n-r}=\pm a_r$ (d) None

3. The number of real and imaginary roots of the equation $x^3 - 6x - 9 = 0$ is _____.

(a) 2 real, 1 imaginary

(b) 3 real, 0 imaginary

(c) 1 real, 2 imaginary

(d) 3 imaginary

4. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then $\alpha^2\beta^2\gamma^2 =$ _____.

(a) $-r$ (b) q^2

(c) r^2 (d) p^2

5. The value of $\sqrt{x^2+16} - \sqrt{x^2+9} =$ _____.

(a) $\frac{8}{x}$ nearly (b) $\frac{8}{2x}$ nearly

(c) $\frac{7}{x}$ nearly (d) $\frac{7}{2x}$ nearly

6. The value of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots =$ _____.

(a) $\log_e 2$ (b) $\log_e\left(\frac{1}{2}\right)$

(c) $\log_{10} 2$ (d) $\log_{10}\left(\frac{1}{2}\right)$

7. $\underset{\theta \to 0}{Lt} \frac{n \sin\theta}{\theta \cos\theta} =$ _____.

(a) n (b) 1

(c) 0 (d) ∞

8. If $x = \cos\theta + i\sin\theta$, then the value of $x^n - \frac{1}{x^n}$ is _____.

(a) $2\sin n\theta$ (b) $2i\sin n\theta$

(c) $2\cos n\theta$ (d) $2i\cos n\theta$

9. The value of $\sinh 2x$ = _____.

(a) $2\sin x \cos x$ (b) $2\sinh x \cos x$

(c) $2\sin x \cosh x$ (d) $2\sinh x \cosh x$

10. The value of $\tanh^{-1}\left(\frac{x^2-1}{x^2+1}\right)$ = _____.

(a) $\log x^2$ (b) $\log x$

(c) $\log 2x$ (d) $\log\sqrt{x}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Find the quotient and remainder when $2x^6 + 3x^5 - 15x^2 + 2x - 4$ is divided by $x + 5$.

Or

(b) Find the relation between the co-efficients in the equation $x^4 + px^3 + qx^2 + rx + s = 0$, in order that the co-efficients of x^3 and x may be removable by the name transformation.

12. (a) Find $\frac{1}{\alpha^5} + \frac{1}{\beta^5} + \frac{1}{\gamma^5}$ when α, β, γ are the roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$.

Or

(b) Solve : $x^3 - 6x - 9 = 0$.

13. (a) Find the sum to infinity of the series

$$\frac{1 \cdot 4}{5 \cdot 10} - \frac{1 \cdot 4 \cdot 7}{5 \cdot 10 \cdot 15} + \frac{1 \cdot 4 \cdot 7 \cdot 10}{5 \cdot 10 \cdot 15 \cdot 20} - \ldots\ldots$$

Or

(b) Evaluate : $\underset{x \to 0}{Lt} \frac{e^x - e^{-x}}{\log(1+x)}$.

14. (a) Expand $\sin^3\theta\cos^5\theta$ in a series of sines multiple of θ.

Or

(b) Find $\underset{\theta\to 0}{Lt}\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$.

15. (a) If $\cos(x+iy)=\cos\theta+i\sin\theta$, then prove that $\cos 2x+\cosh 2y=2$.

Or

(b) Separate into real and imaginary parts of $\tanh(1+i)$.

PART C — ($5\times 8=40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Solve : $6x^5-x^4-43x^3+43x^2+x-6=0$.

Or

(b) Show that the equation $x^4-3x^3+4x^2-2x+1=0$ can be transformed into a reciprocal equation by diminishing the roots by unity. Hence solve the equation.

17.

17. (a) If $\alpha, \beta, \gamma, \delta$ the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, then find (i) $\sum \alpha^2$ (ii) $\sum \alpha^2 \beta\gamma$ (iii) $\sum \alpha^2 \beta^2$ (iv) $\sum \alpha^3 \beta$.

Or

(b) Find the positive roots of the equation $x^3 - 3x + 1 = 0$ correct to three places of decimals.

18. (a) Sum the series

$$1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \ldots \text{to } \infty .$$

Or

(b) Show that

$$1 + \left(\frac{1}{2} + \frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right)\frac{1}{4^2} + \left(\frac{1}{6} + \frac{1}{7}\right)\frac{1}{4^3} + \ldots = \log\sqrt{12} .$$

19. (a) Prove that the equation $\frac{ah}{\cos\theta} - \frac{bk}{\sin\theta} = a^2 - b^2$ has four roots and that the sum of the four values of θ is equal to an odd multiple of π radians.

Or

(b) Expand $\tan 4\theta$ interms of $\tan\theta$ and show that $\tan\frac{\pi}{16}$, $\tan\frac{5\pi}{16}$, $\tan\frac{9\pi}{16}$, $\tan\frac{13\pi}{16}$ are the roots of the equation $x^4+4x^3-6x^2-4x+1=0$.

20. (a) If $\log\sin(\theta+i\phi)=L+iB$, then prove that $2e^{2L}=\cosh 2\phi-\cos 2\theta$.

Or

(b) Prove that

(i) $\sinh^{-1}x=\log_e\left(x+\sqrt{x^2+1}\right)$

(ii) $\cosh^{-1}x=\log_e\left(x+\sqrt{x^2-1}\right)$

(iii) $\tanh^{-1}x=\frac{1}{2}\log_e\left(\frac{1+x}{1-x}\right)$.